Complex Reflection Coefficients Applied to Steep Sloping Structures

Supplement to:
„Phase Jump due to Partial Reflection of Irregular Water Waves at Steep Slopes“,
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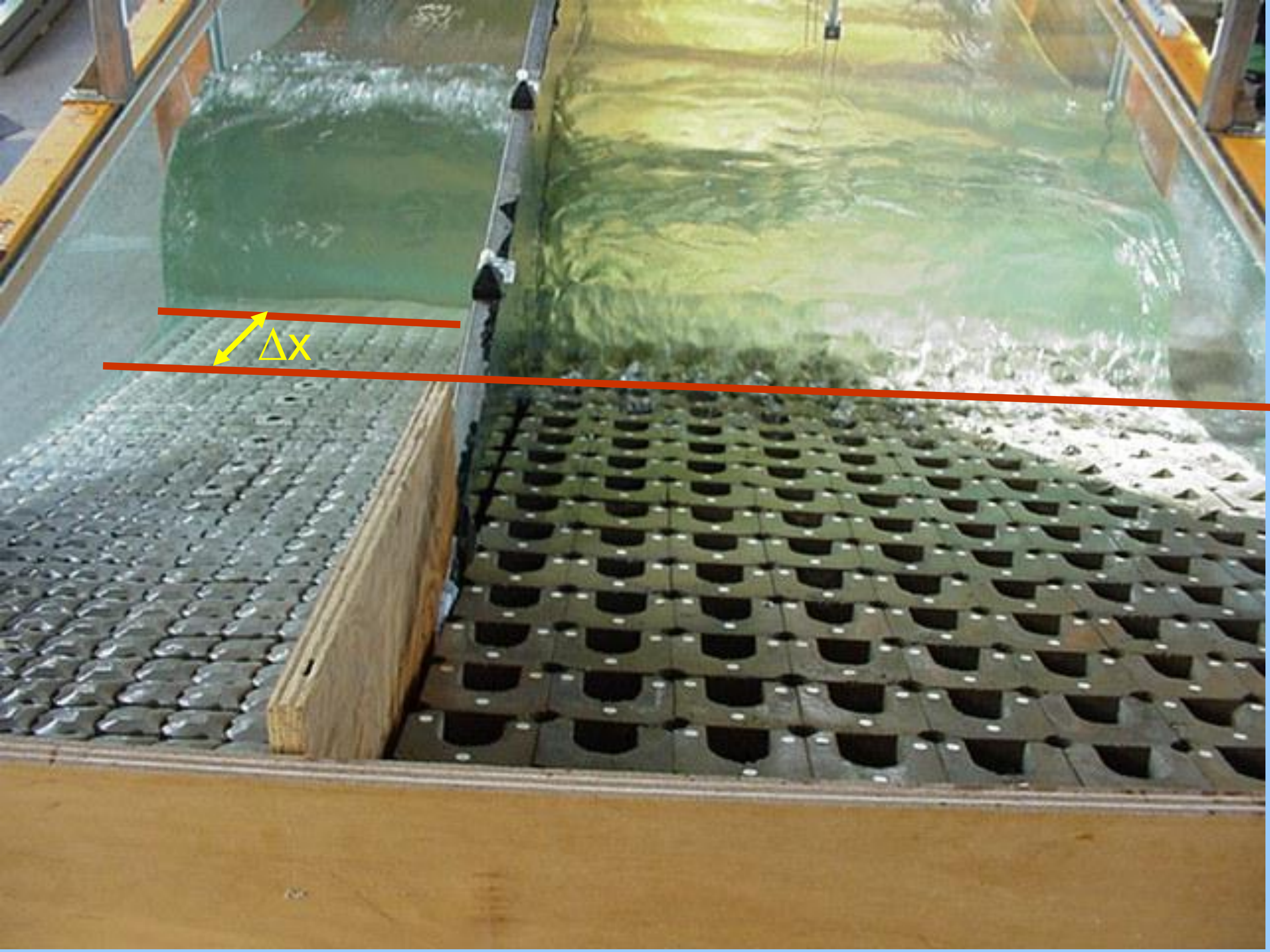
Scope of Presentation

- **Analogue:**
  P**hase shift** $\Delta\varphi$ between incident and reflected waves marks cases of ***positive and negative*** **partial reflection** at coastal structures.

- **Theoretical derivation of the**
  Complex reflection coefficient $\Gamma = C_r e^{i\Delta\varphi}$

- **Presentation of Measurements:**
  Magnitudes and Phases, Re[Γ] and Im[Γ] parts of the Complex reflection coefficient (CRC).

- Discuss influence on Types of Breakers



Tests on the stability of Hollow Cubes

Model scale: 1:5

Slope: 1:n = 1:3

Wave heights up to H = 0.35m

Punging breaker on quasi Smooth Slope

$C_r = 0.33;\ \Delta\varphi \approx 216°$

Collapsing breaker on Hollow Cubes

$C_r = 0.20;\ \Delta\varphi \approx 163°$

$$\Delta x \approx 0.15m$$

Monochromatic Waves:
Total wave field = incident + reflected wave

$$y(x,t) = Ae^{i(\omega t - kx)} + C_r Ae^{i(\omega t + kx + \Delta\varphi)}$$

$$= (e^{-ikx} + C_r e^{i\Delta\varphi} e^{ikx}) Ae^{i\omega t} = (e^{-ikx} + \Gamma e^{ikx}) Ae^{i\omega t}$$

Complex reflection coefficient $\boldsymbol{\Gamma = C_r e^{i\Delta\varphi}}$

considering magnitude $C_r = A_r/A_i = H_r/H_i$ and phase shift $\Delta\varphi$

Applying Euler´s formula, two special cases:

***Positive* Total Reflection** where $\Delta\varphi = 0°$ and $C_r = 1$ → $\Gamma = 1$

$$y(x,t) = \left(e^{ikx} + e^{-ikx}\right) Ae^{i\omega t} = 2\,A\cos kx\; e^{i\omega t}$$

→ perfect standing wave **without** phase jump (Clapotis)

***Negative* Total Reflection** where $\Delta\varphi = 180°$ and $C_r = 1$ → $\Gamma = -1$

$$y(x,t) = \left(e^{-ikx} - e^{ikx}\right) Ae^{i\omega t} = -2\,i\,A\sin kx\; e^{i\omega t}$$

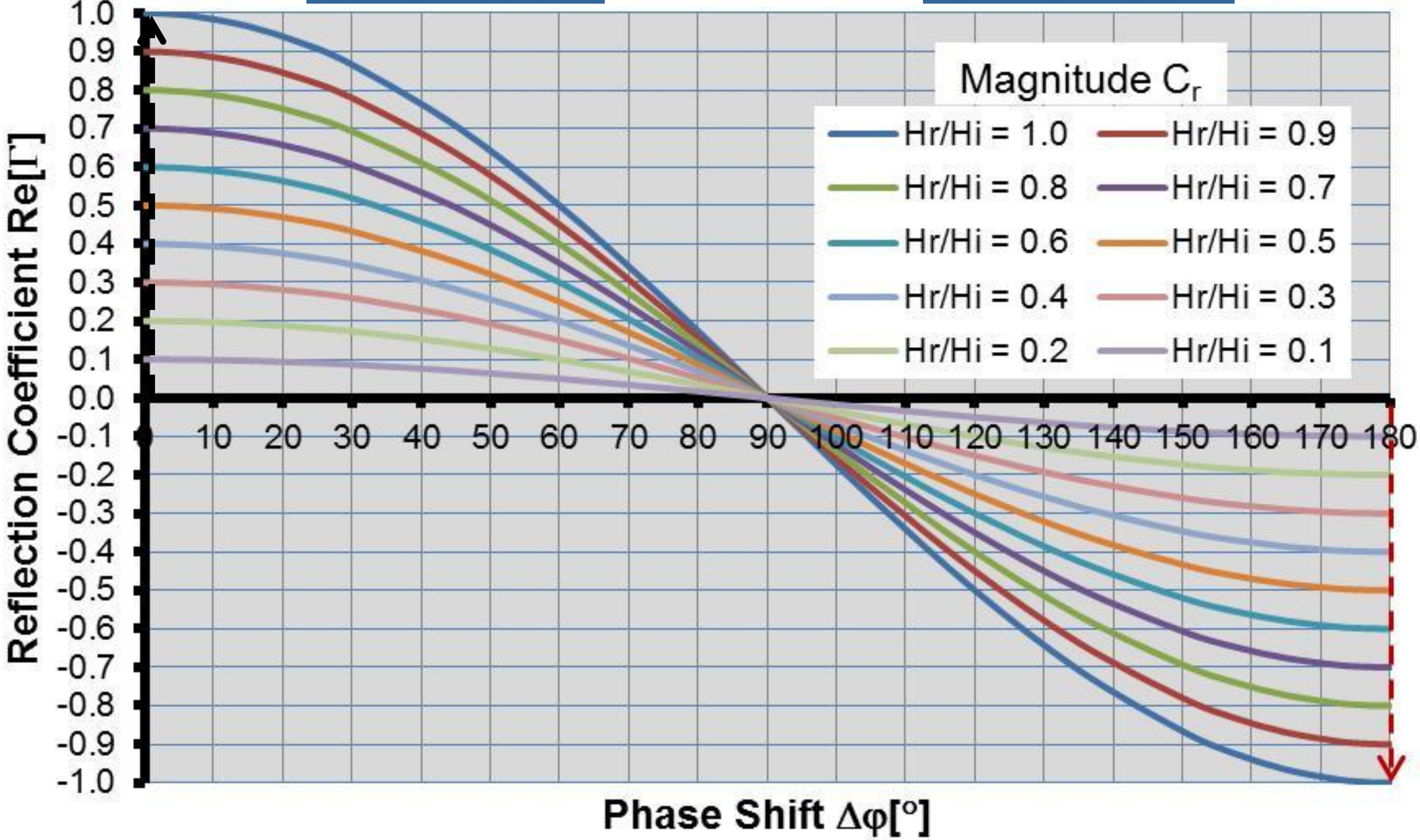
→ perfect standing wave **with** phase jump $\Delta\varphi = 180°$ (Clapotis)

Real part of the *komplex* reflection coefficient

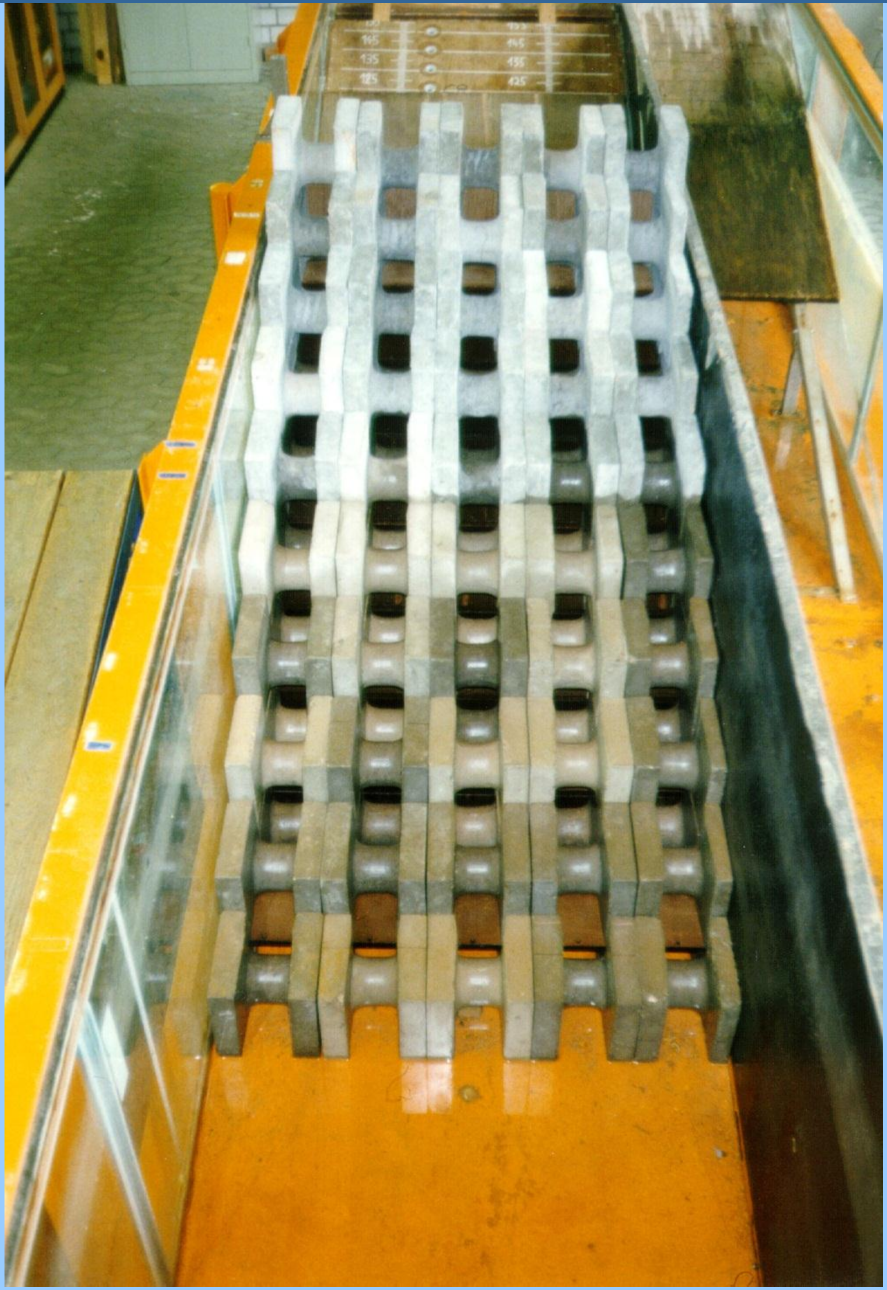
$$\Gamma = C_r e^{i\Delta\varphi}$$

positive reflection

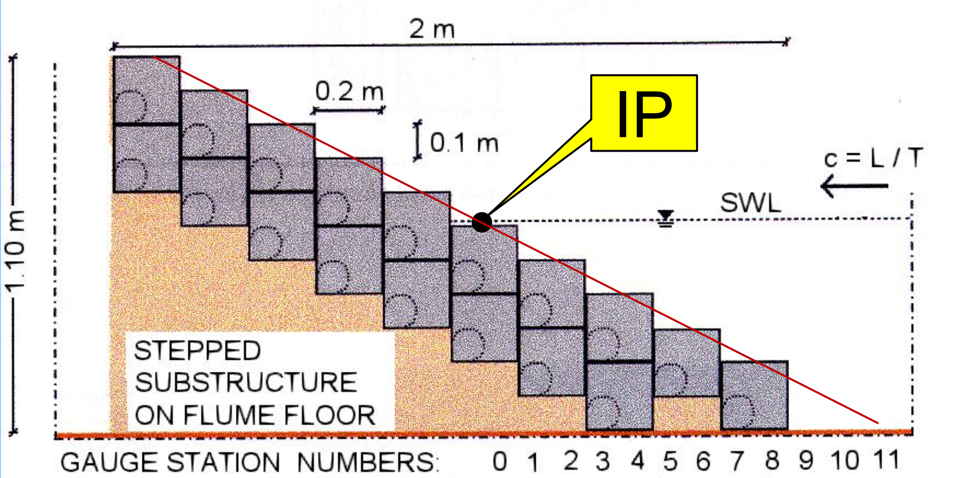
negative reflection



Hollow Cubes versus smooth slope structure 1:2



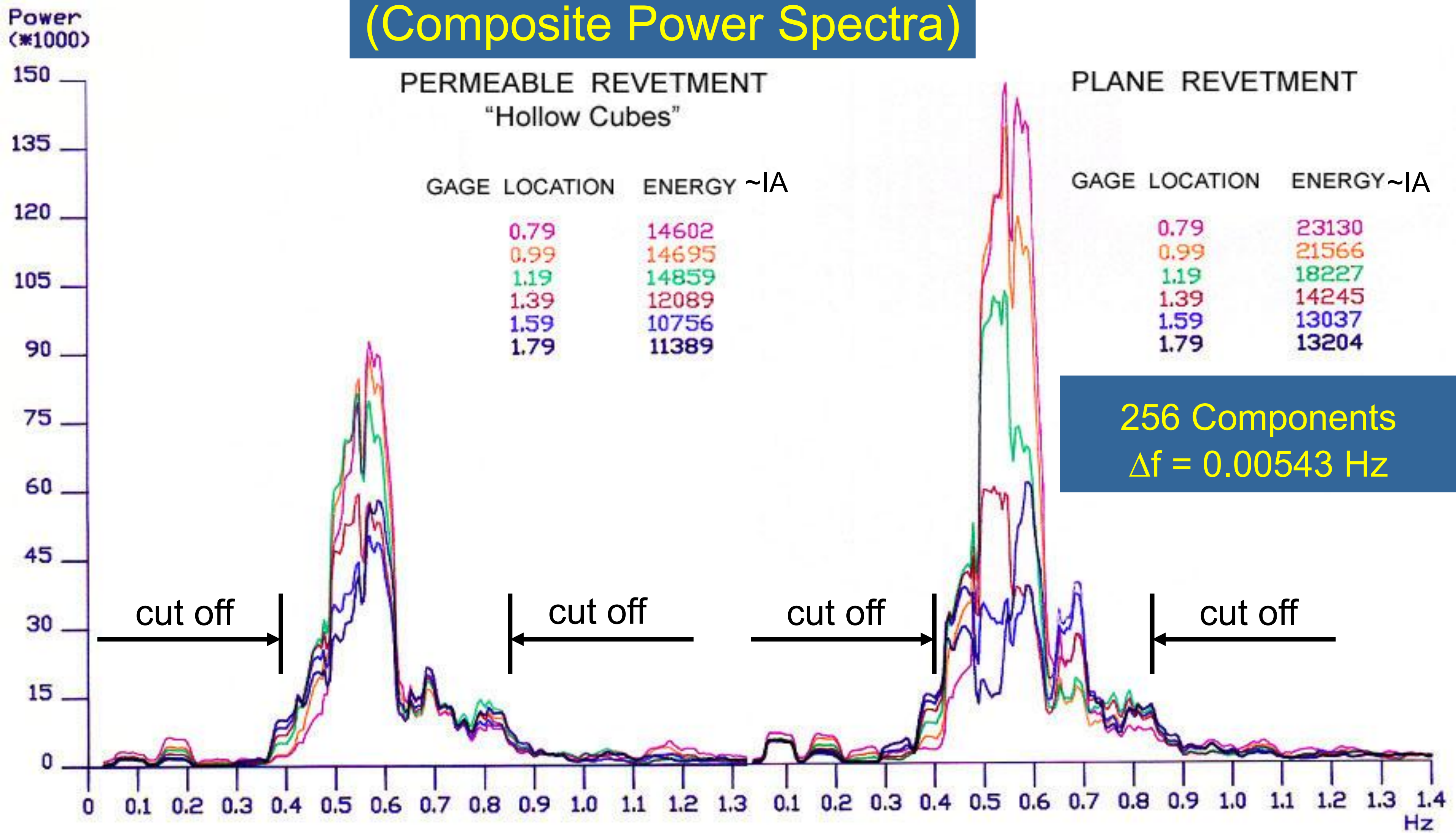
Hollow Cubes piled up to form a **stepped face hollow seawall** structure (2-layer-system).
Slope: 1:2
Model scale: 1:10



Hollow Cubes	$C_r \approx 0.2$; $\Delta\varphi \approx -20°$
Smooth slope	$C_r \approx 0.7$; $\Delta\varphi \approx 154°$

Energy Density Spectra of Water Level Deflections

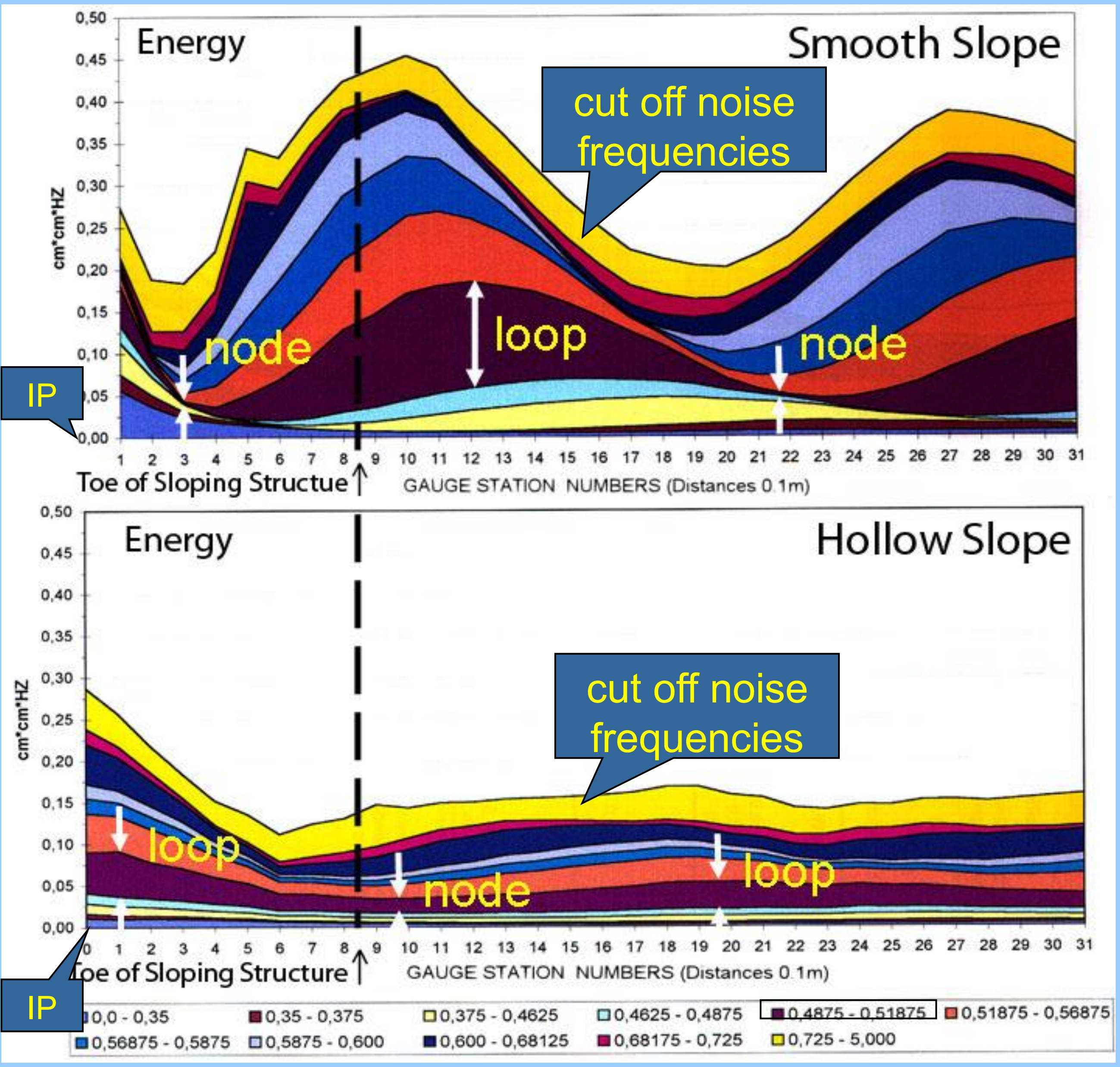
(Composite Power Spectra)



Permeable revetment **Plane revetment**

(measured synchronously at slopes 1:3, distorted by re-reflection)

Energy in front of slopes 1:2



Comments:

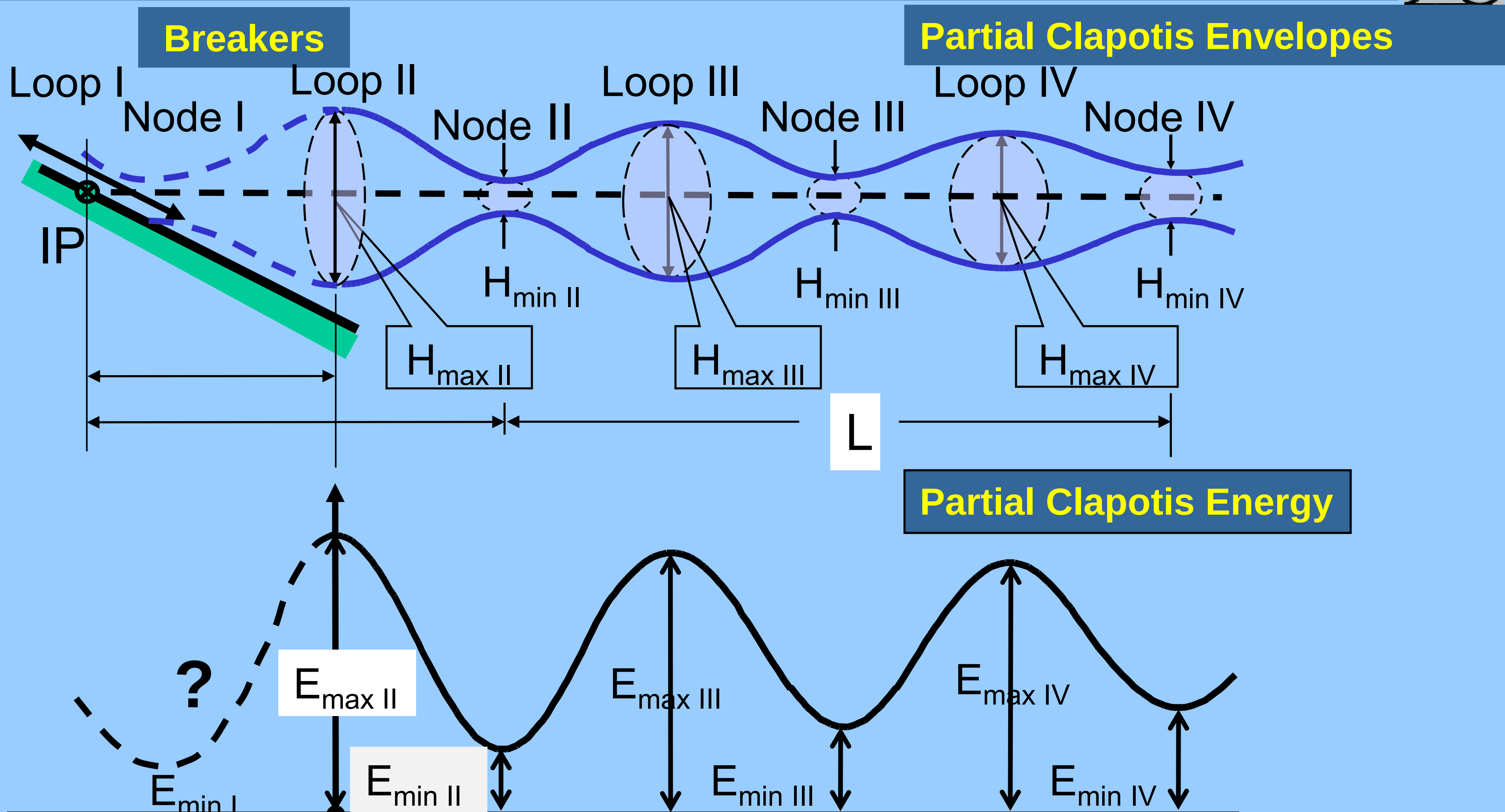
- Integrated spectrum area values (E) plotted with distance from IP:
  →partial Clapotis L≈3.8m

- Total Energy distributed to subfrequency ranges mark **10 component partial clapotis waves**.
  E_{max} = loop, E_{min} = node

- The higher the frequency of partial clapotis waves the more they are shifted in upslope direction.

Close to IP
- **nodes** at **smooth** slope and
- **loops** at **hollow** slope.

General properties of partial standing waves at a slope



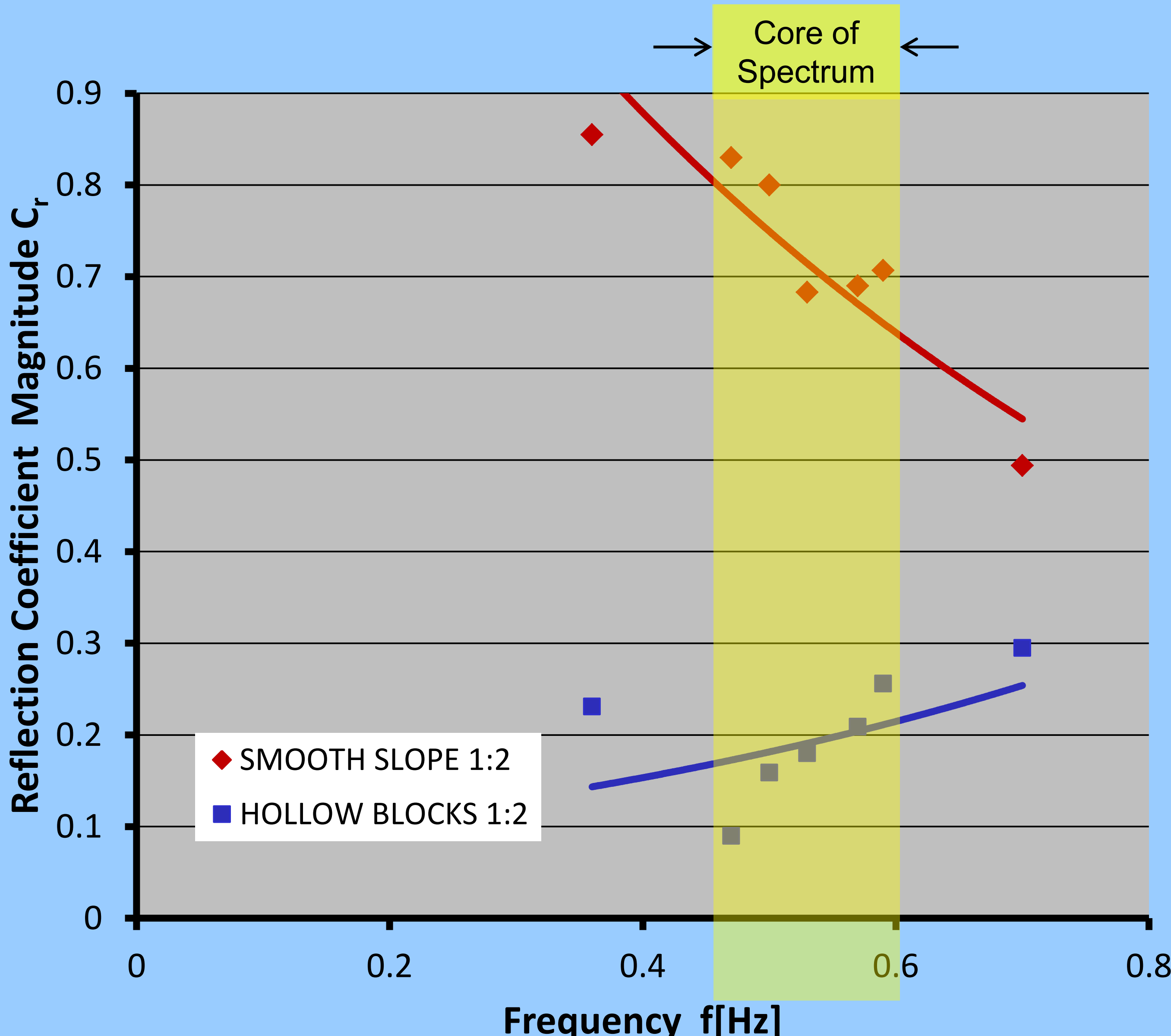
CRC Magnitude

$$C_{r,i} = \frac{\sqrt{E_{\max,i}} - \sqrt{E_{\min,i}}}{\sqrt{E_{\max,i}} + \sqrt{E_{\min,i}}}$$

where

$E_{max,i}$ = maximum energy at loop i
and
$E_{min,i}$ = minimum energy at node i

Reflection Coefficient Magnitudes C_r at slopes 1:2

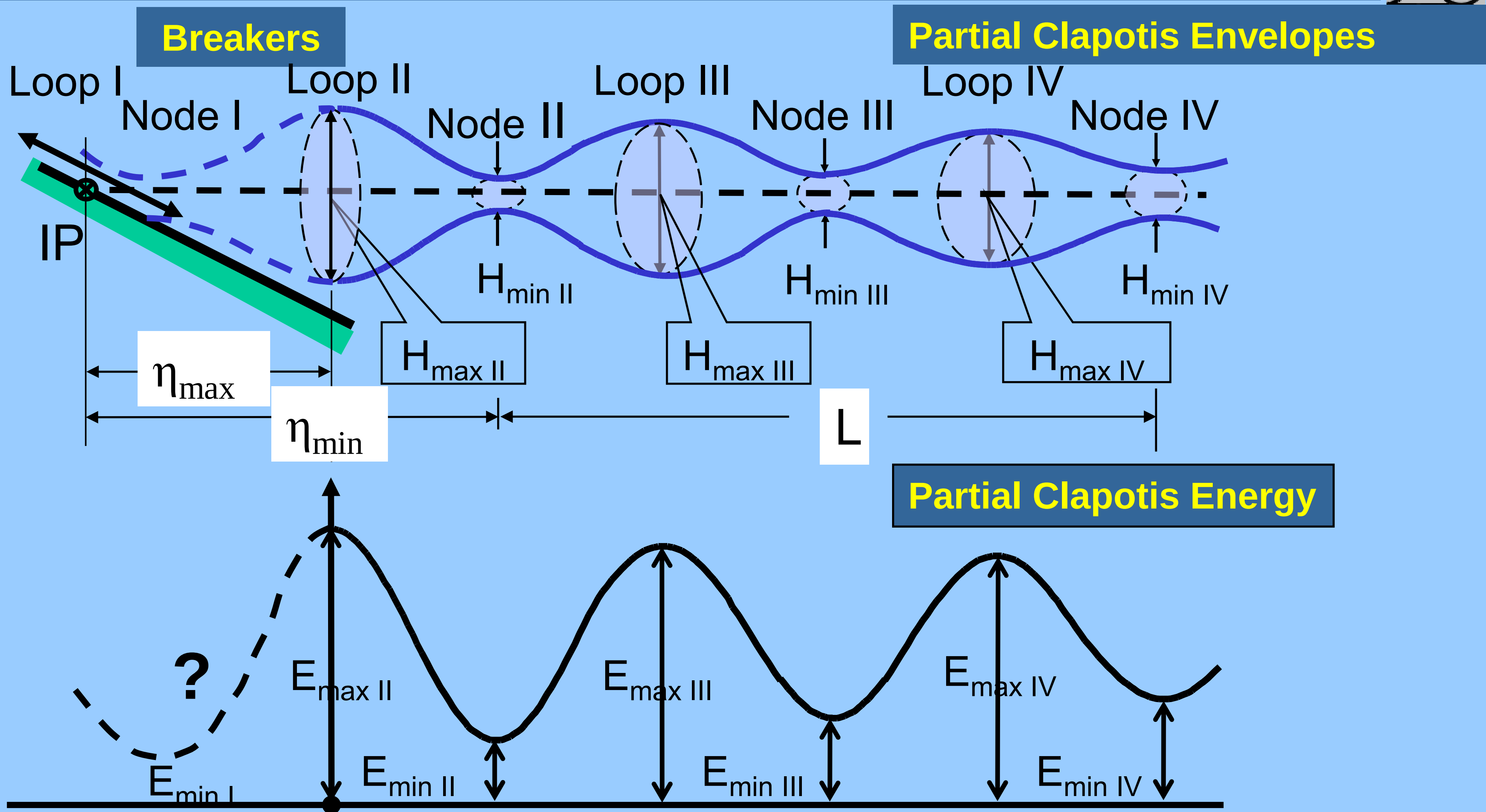


smooth slope

5 component partial standing waves representing the core of the spectrum

hollow slope

General properties of partial standing waves at a slope

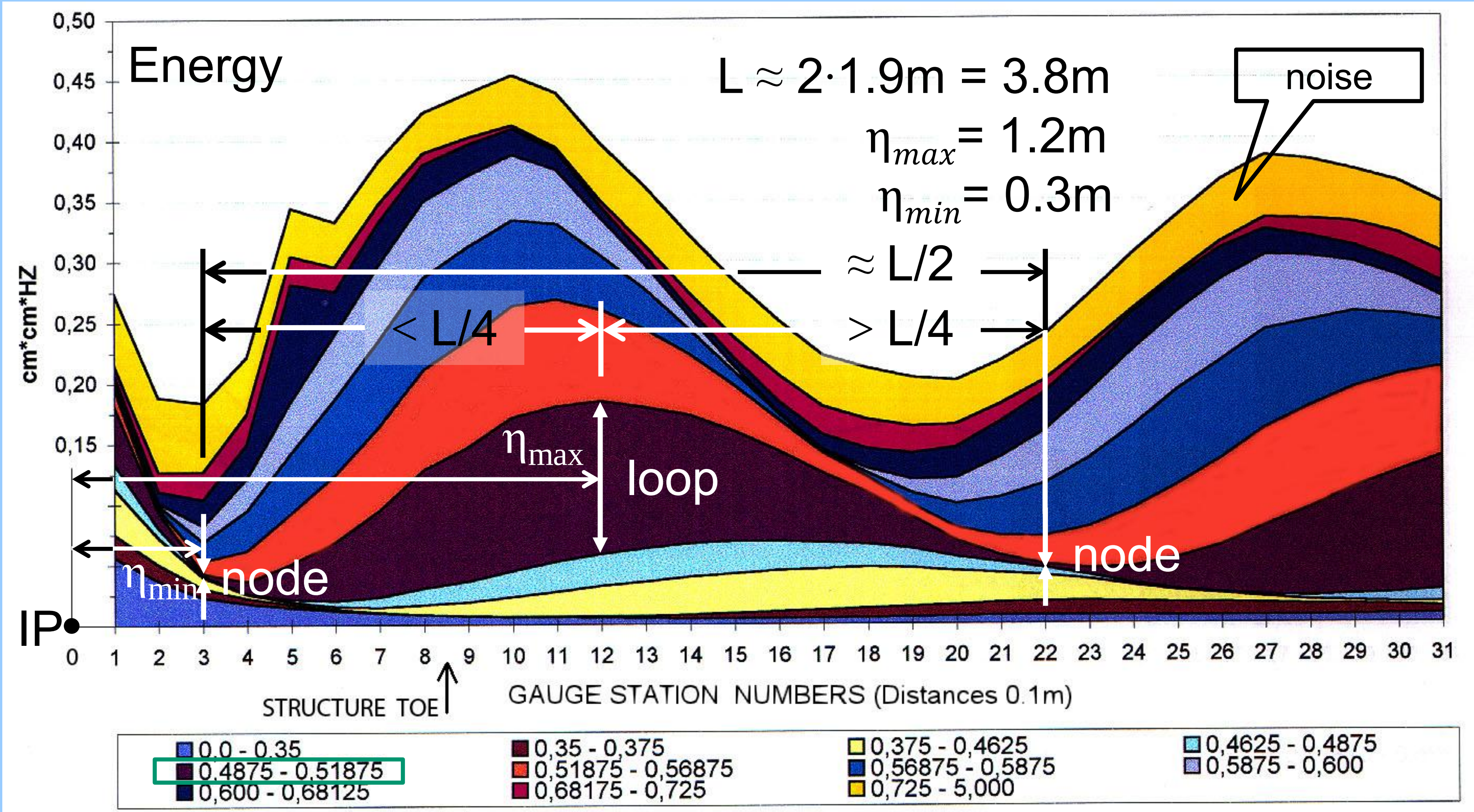


CRC Phase

Loop nearest to IP:
$\Delta\varphi[°]=360(1-2\eta_{max}/L)$

Node nearest to IP:
$\Delta\varphi[°]=180(1-4\eta_{min}/L)$

Phase shifts Δφ of component partial standing waves



Loop:

$\Delta\varphi = 360(1 - 2\eta_{max}/L)$
$= 132.6°$

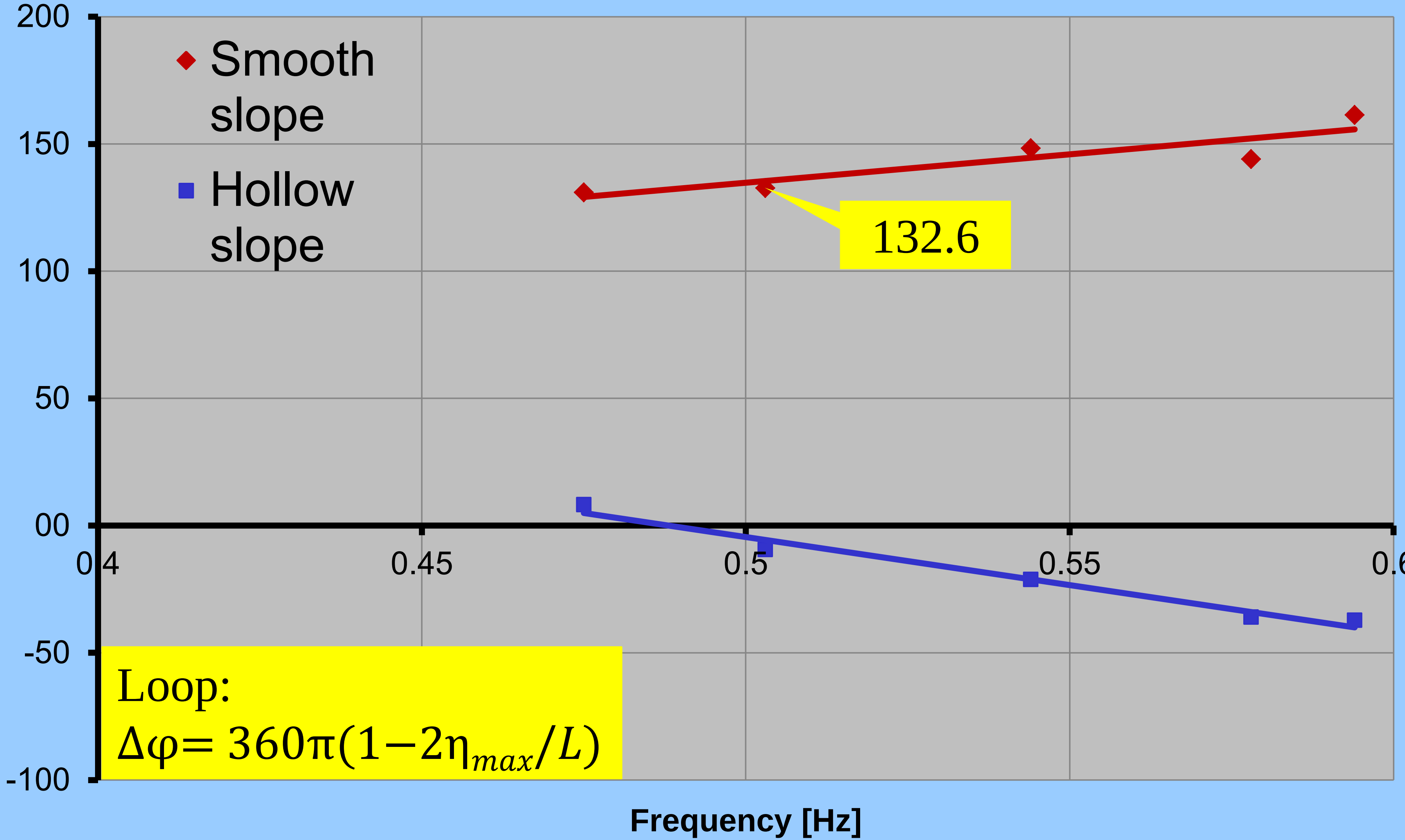
Horizontal wave assymmetry:

$\eta_{max} - \eta_{min} \approx L/4$

Node:

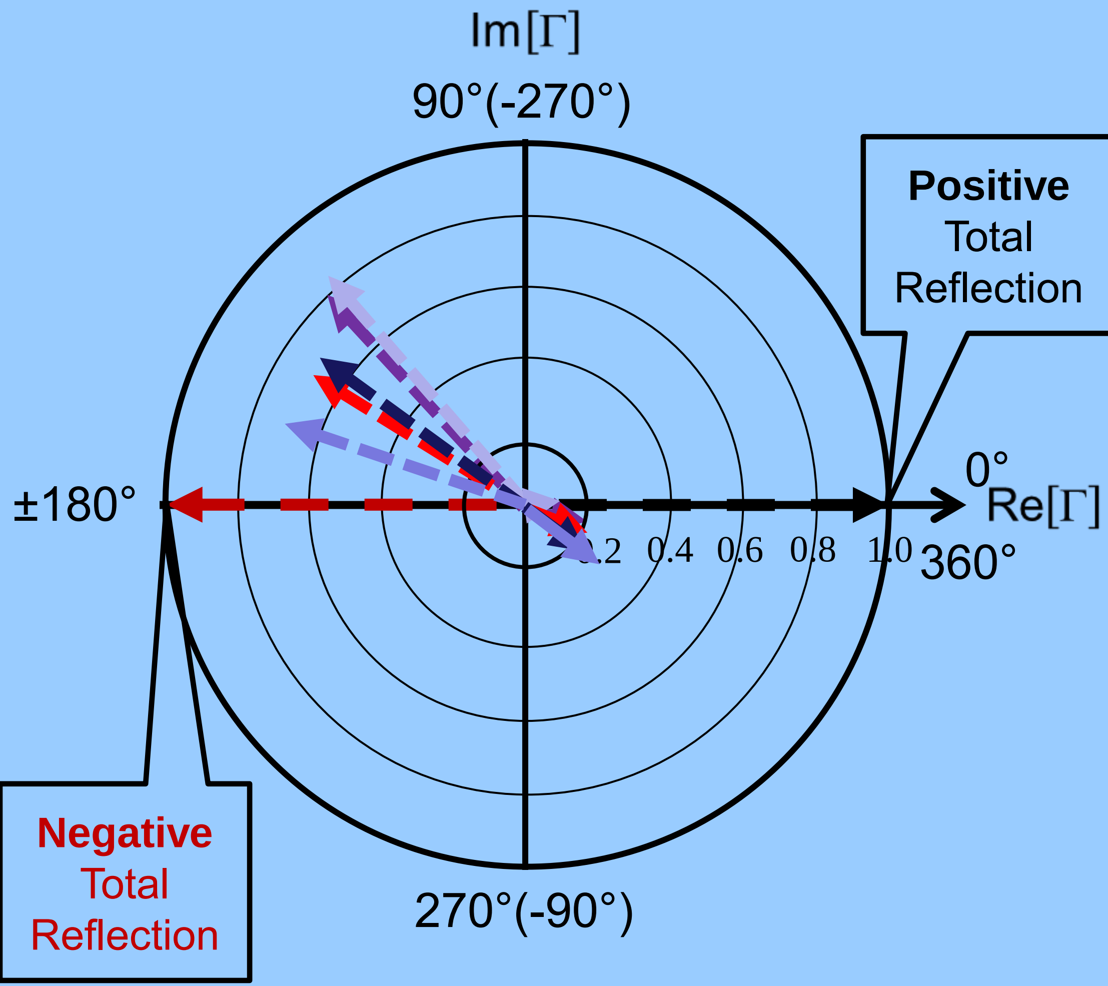
$\Delta\varphi = 180(1 - 4\eta_{min}/L)$
$= 123.2°$

Phase shifts $\Delta\varphi[°]$ at slopes 1:2



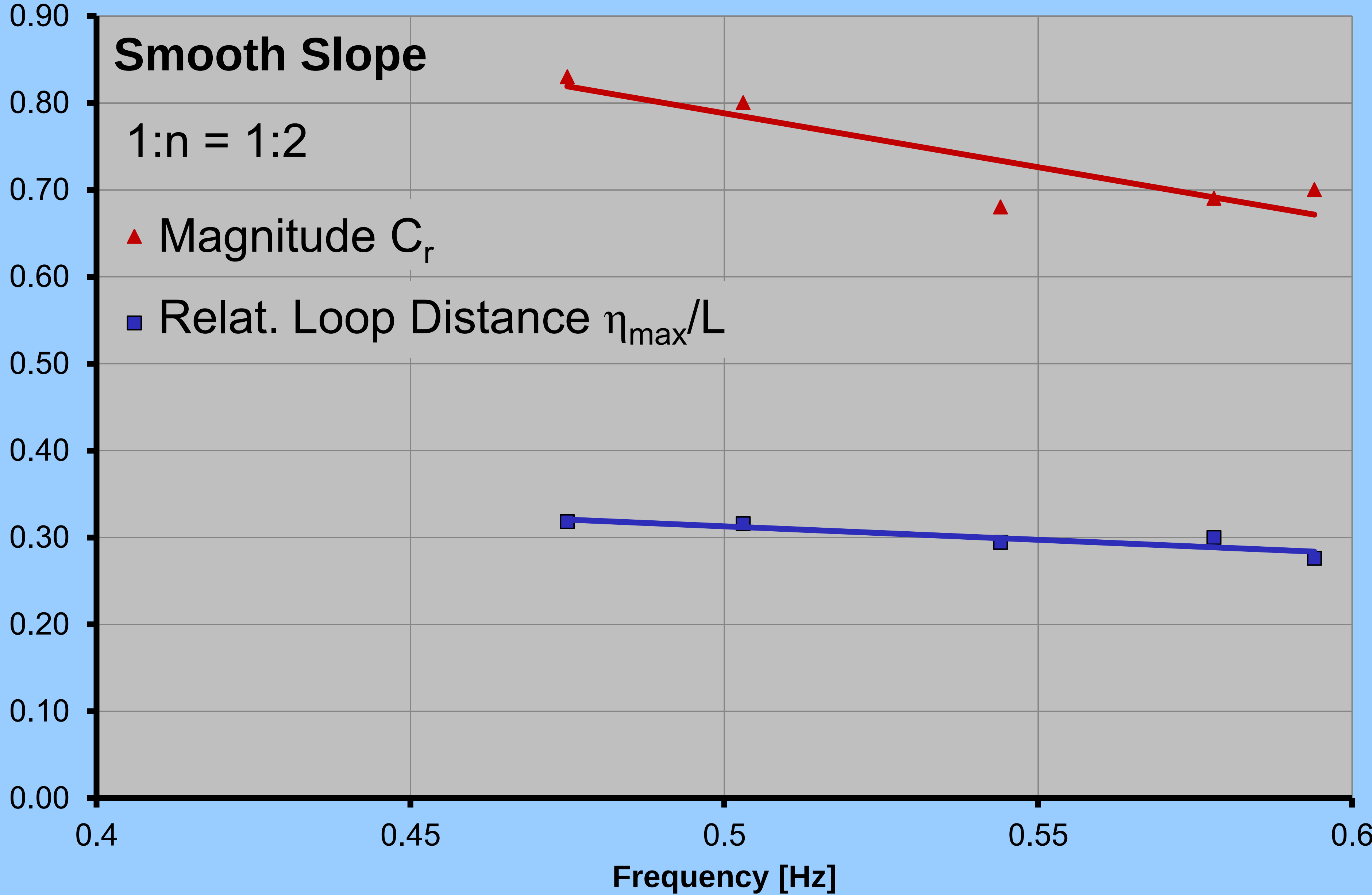
Complex Reflection Coefficients $\Gamma = C_r e^{i\Delta\varphi}$ at slopes 1:2

Gaussian Phasor Diagram



f[Hz]	L[m]	η_{max}[m]	C_r	$\Delta\varphi$[°]
Smooth Slope				
0.475	4.4	1.40	0.83	130.9
0.503	3.8	1.20	0.80	132.6
0.544	3.4	1.00	0.68	148.2
0.578	3.0	0.90	0.69	144.0
0.594	2.9	0.80	0.70	161.4
Hollow Cubes				
0.475	4.4	2.15	0.09	8.2
0.503	3.8	1.95	0.16	-9.5
0.544	3.4	1.80	0.18	-21.2
0.578	3.0	1.65	0.21	-36.0
0.594	2.9	1.60	0.25	-37.2

Magnitudes C_r and relative loop distances η_{max}/L



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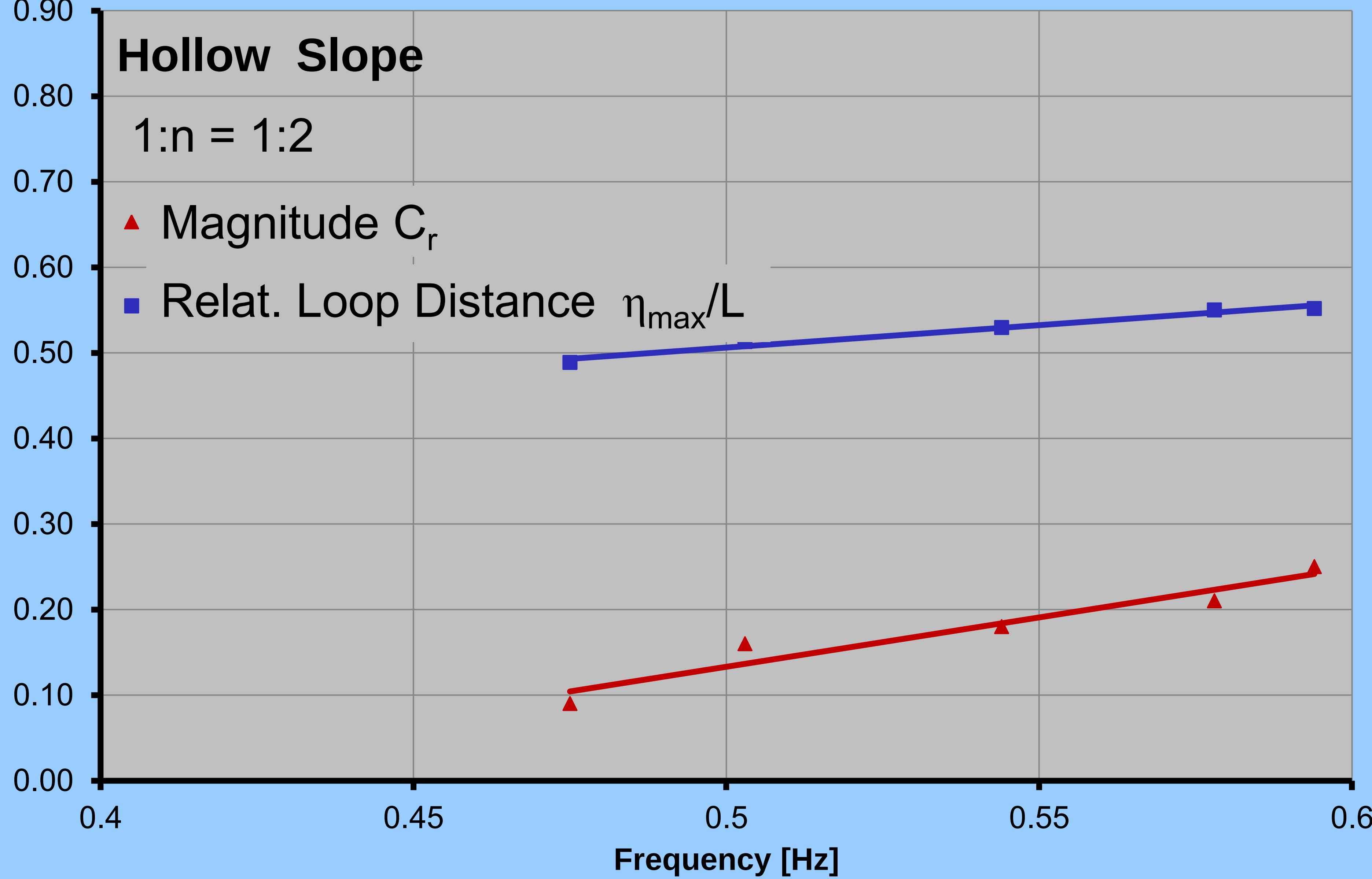
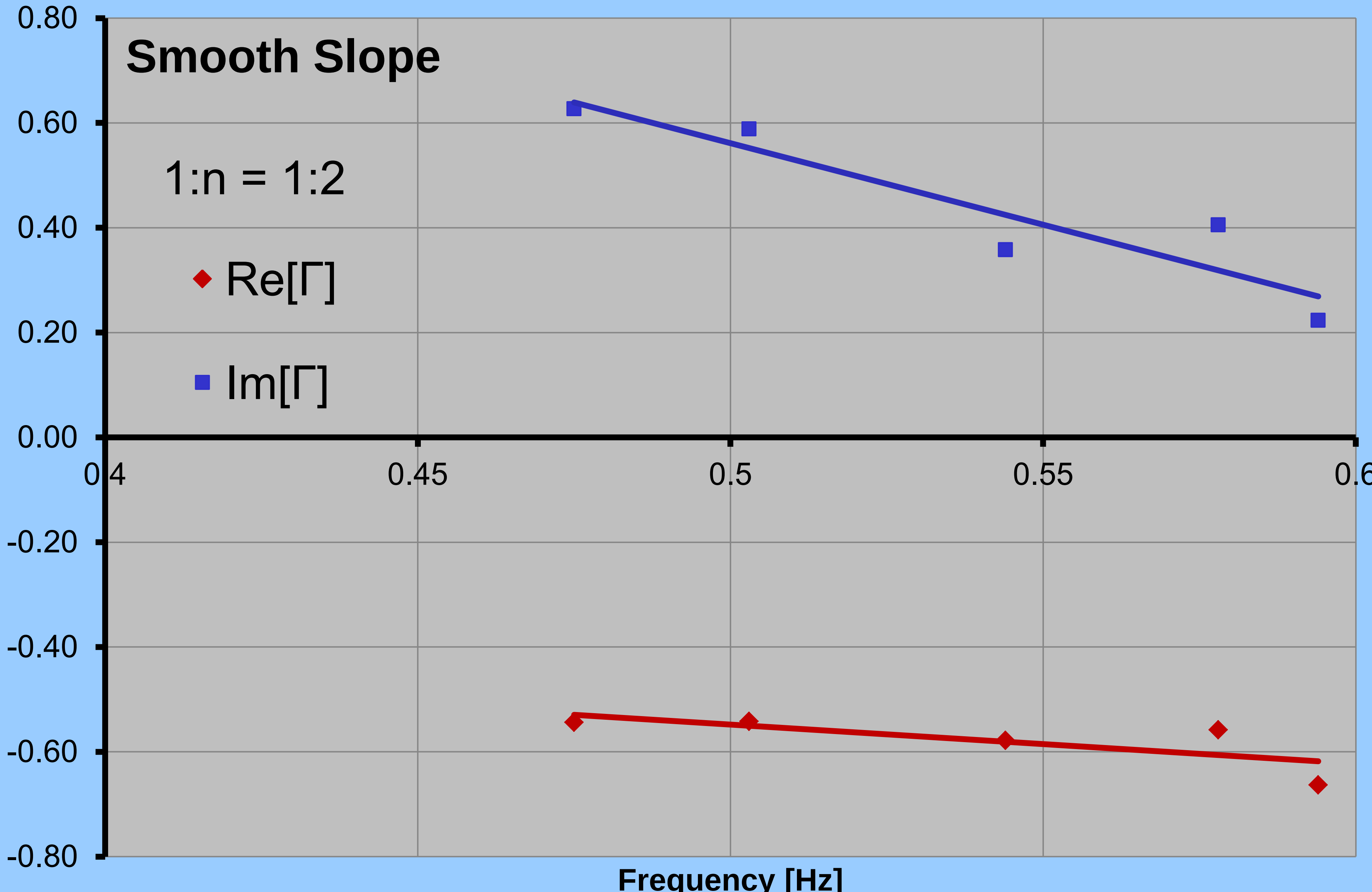


Real and imaginary parts of CRC $\Gamma = C_r e^{i\Delta\varphi}$



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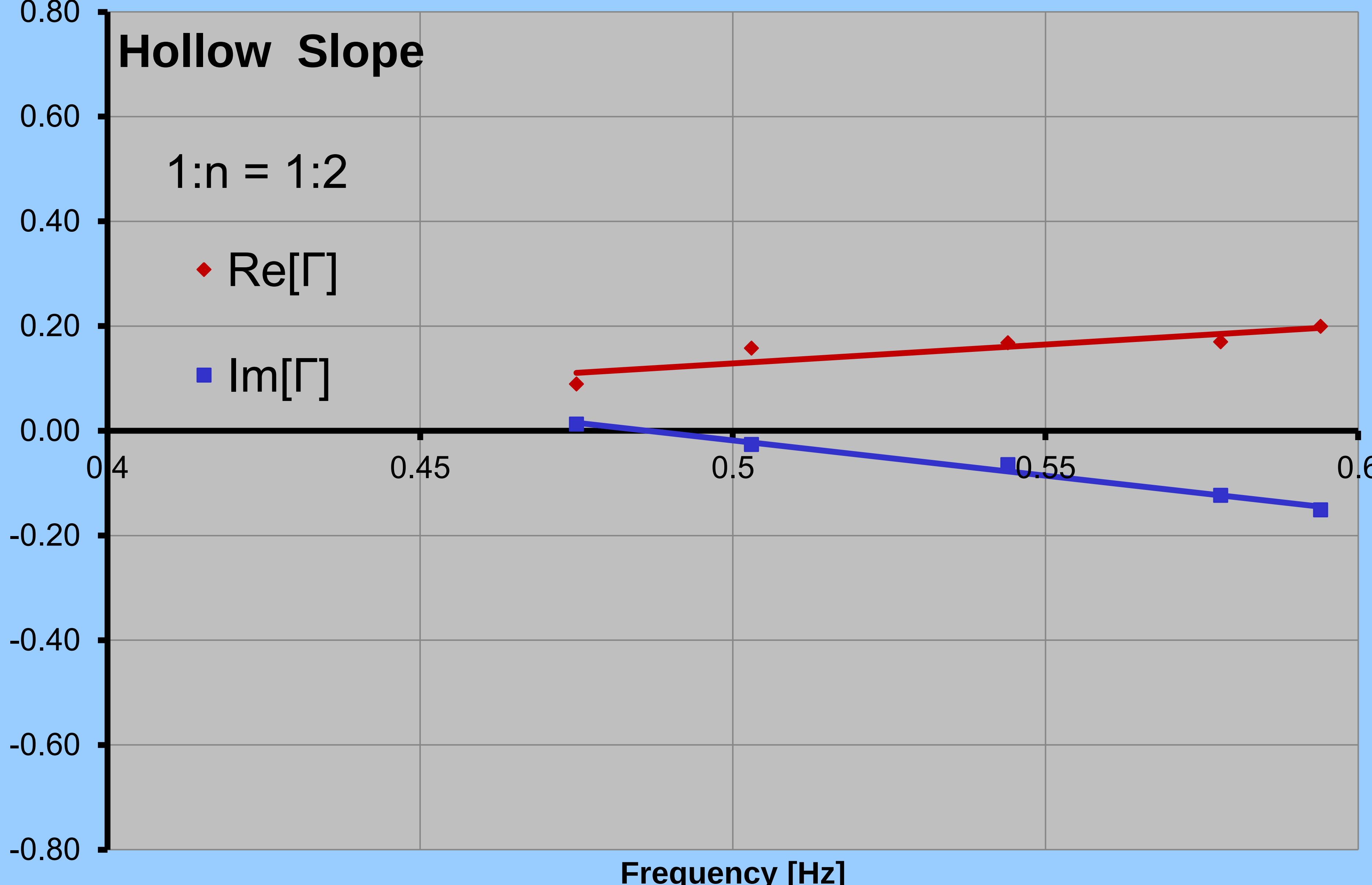
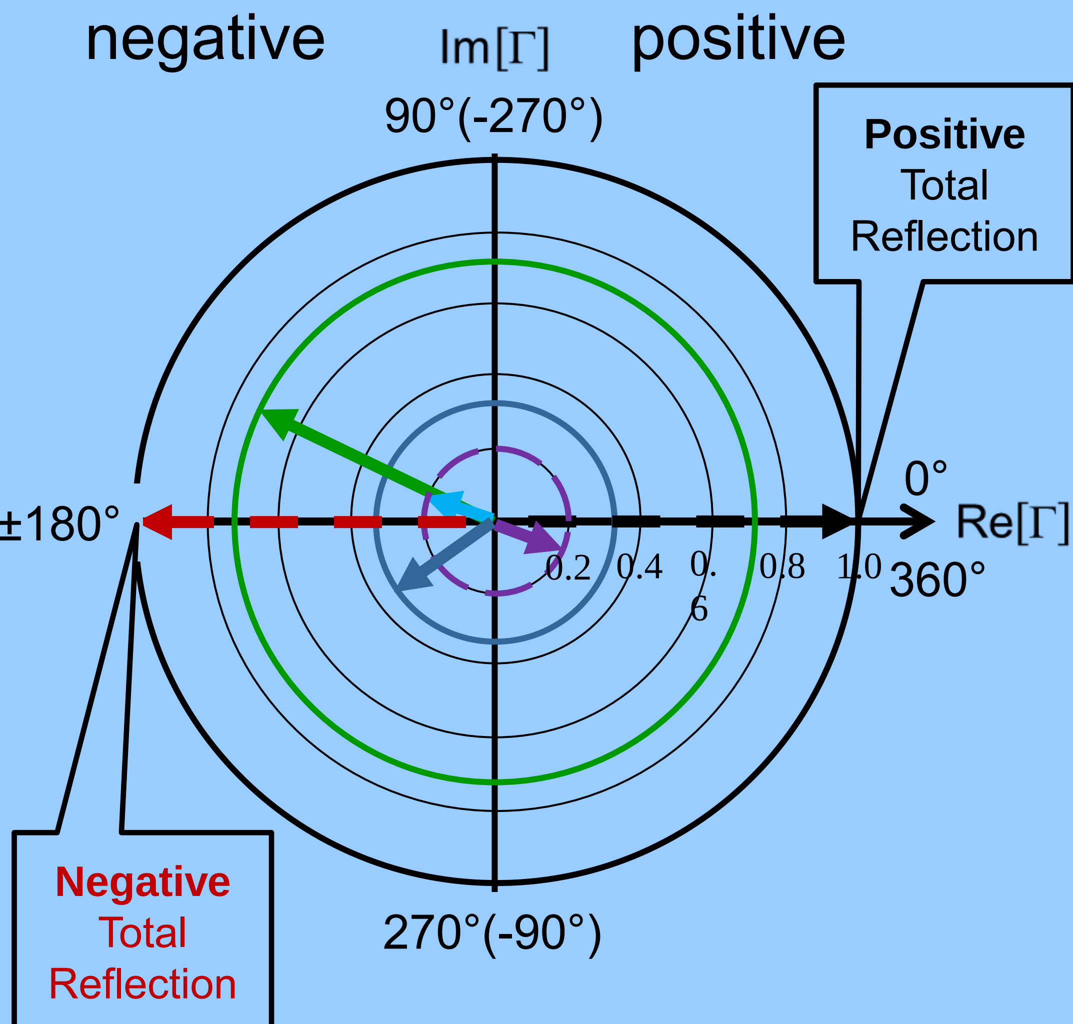


Average Complex Reflection Coefficients $\Gamma = C_r\, e^{i\Delta\varphi}$

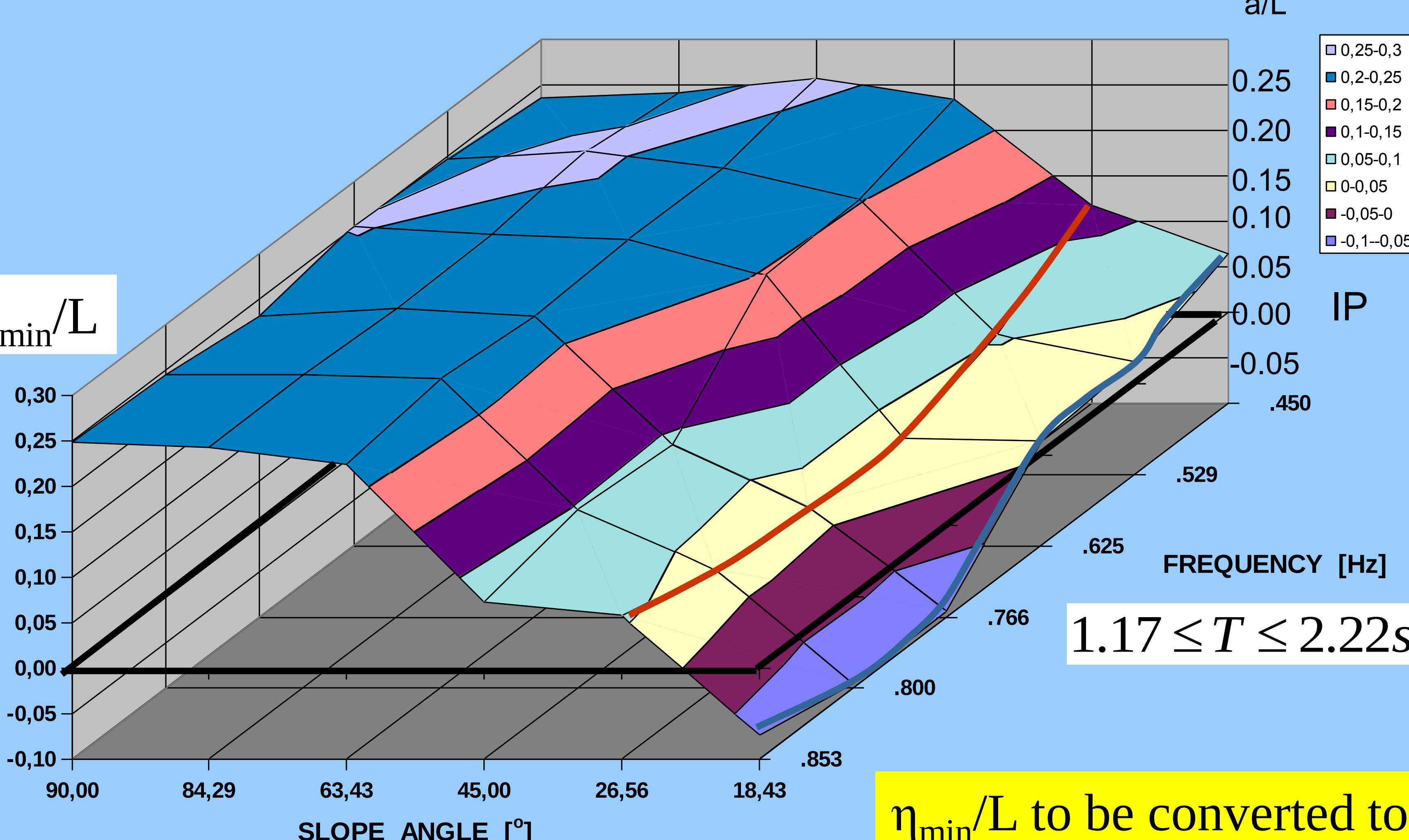
Gaussian Phasor Diagram



Different revetments on slopes 1:3 and 1:2

f[Hz]	L[m]	η_{max}[m]	C_r	Δφ[°]
Smooth slope, 1:3				
0.38	3.65	0.73	0.33	216.0
to	Hollow revetment, 1:3			
0.76	3.65	1.00	0.20	162.7
Smooth slope, 1:2				
0.36	3.5	1.00	0.72	154.3
to	Hollow Cubes, 1:2			
0.70	3.5	1.85	0.20	-20.6

Monochromatic waves at **smooth** slopes (1995): Relative node distances η_{min}/L with slope angle and frequency



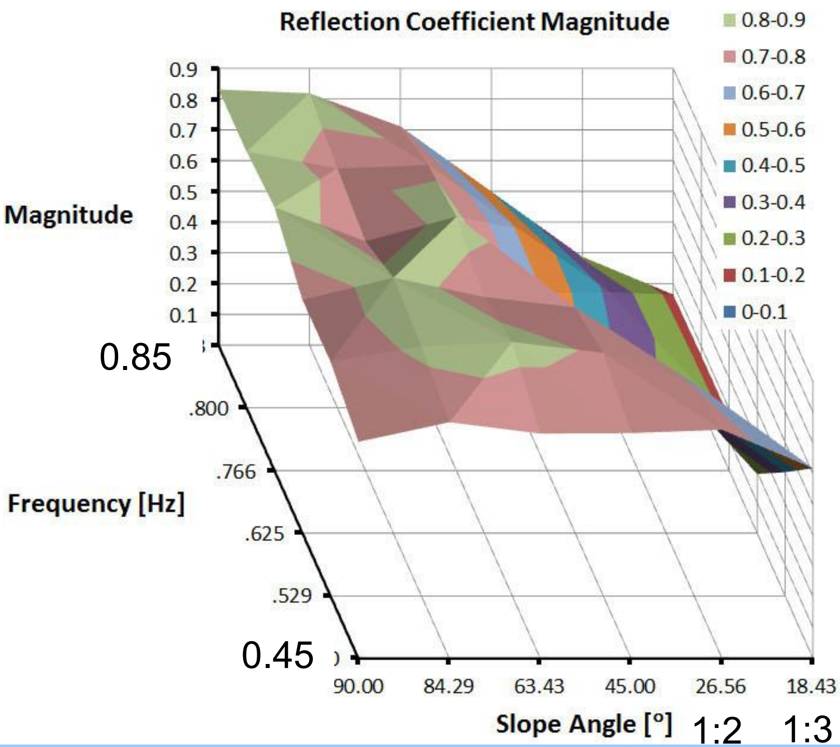
$1.17 \leq T \leq 2.22s$

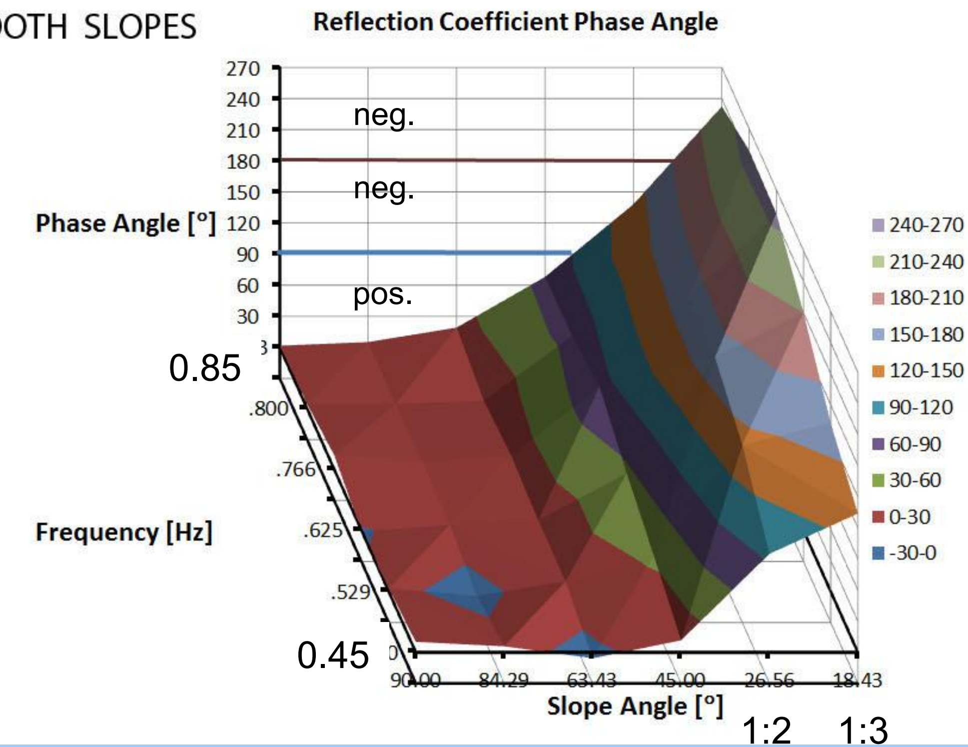
tanα = 1:m   1:0.1   1:0.5   1:1   1:2   1:3

η_{min}/L to be converted to $\Delta\varphi[°]=180(1-4\eta_{min}/L)$

Magnitudes and Phases of the CRC plotted with wave frequencies 0.45 – 0.85 Hz and slope angles 90° - 18°

SMOOTH SLOPES





Results of monochromatic waves:

With respect to both axis (frequency and slope angle) there are

- **<u>opposite</u> trends of magnitudes and phase angles**,
  - Trend decreasing for longer waves and
  - Trend increasing for flatter slopes.

Types of breakers correlating to CRC ?

Phase angle $\Delta\varphi$ controls the positioning of the partial standing wave at the structure and consequently the location of the *breaker depth*.

Conclusion: Phase angle $\Delta\varphi$ is **needed** for a complete description of the breaking at a slope.

Presumptions on types of breakers correlating to phase angles:

Phase angle $\Delta\varphi$	Type of breaker	Further condition
≈ 0°	broken **Clapotis**	super critical steepness
1st or 4th quadrant (positive reflection)	no distinct breaker type	dissipation>transmission
≈ 180°	**surging** breaker	low dissipation
2nd or 3rd quadrant (negative reflection)	collapsing or plunging breaker	dissipation and transmission

Measurements too few for more consistent statements !

Trivial suggestions on the **joint effect of reflection, transmission and dissipation** of breaking waves at a slope

Verify current findings in a **natural** scale:
Specify phase shifts for **longer waves** and **less inclined** slopes.

Consider the phase difference $\Delta\varphi$ in the presentations of both

- the reflection coefficients and
- the types of breakers

as functions of the **Iribarren** Number